

EXAMINATION 07-11-2013

ATOMS AND MOLECULES

YOU CAN MAKE USE (IF YOU THINK YOU HAVE TO) OF THE FOLLOWING FORMULA'S:

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

$$g_F = \frac{F(F+1) - I(I+1) + J(J+1)}{2F(F+1)} g_J$$

1. ENERGY LEVELS (25 POINTS)

In Figure 1, on the next page, you see two sets of electronic energy levels. The energy levels are indicated with lines. Not all levels are given, just the most important ones. They are roughly, but not exactly, drawn to scale (with energy on the vertical axis). One of the two is a one-electron system, the other one is a two-electron system. Indicate the following in these figures (either on your own paper, or on the question sheet):

- A. Which one is the two-electron system, and which one is the one-electron system?
- B. The two-electron system is Calcium. From your knowledge of the periodic table of elements, give the electron configuration of the Calcium atom.
- C. Label the energy levels of the Calcium system with term symbols, and also indicate the electron configuration (only the changing part) for these energy levels.
- D. Indicate a strong allowed transition and an inter combination line in one of these energy level schemes.
- E. The set of three lines close together on the left-figure are split by spin-orbit coupling. The spin-orbit coupling constant in this case is 52 cm^{-1} . Use the interval rule, and your knowledge of the quantum numbers for this case, to calculate the spacing between these three levels.
- F. Consider the light from a glowing gas of the element on the right. This light is observed by an old spectrometer (that can not resolve the small effects caused by the nuclear spin). In how many components does the line between level A and B split if a weak magnetic field is applied? Calculate the energy shift of each line, with respect to the zero-magnetic field case, expressed in units of $\Delta = \mu_B B$.
- G. An odd isotope of this element has a nuclear spin $I = 3/2$. Because of this, level C has hyperfine splitting. Draw the splitting schematically, label the hyperfine states with their F quantum number and express the energy separation between these states in terms of the hyperfine constant A .

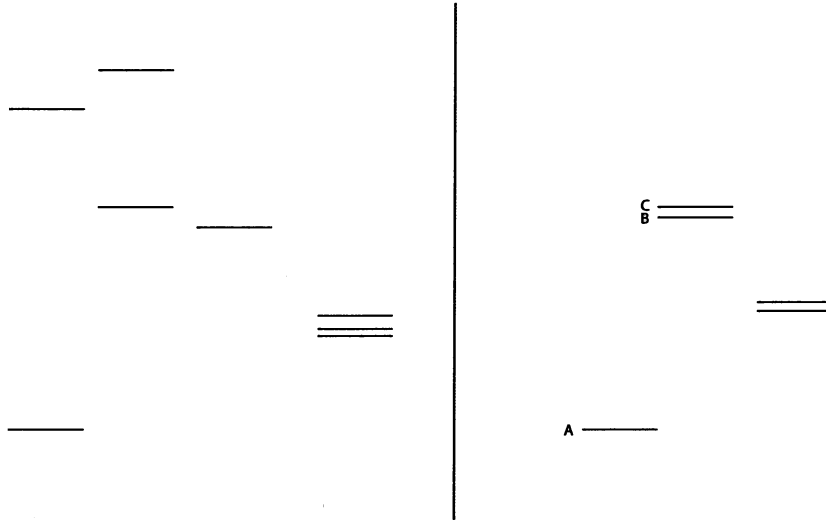


FIGURE 1. Two sets of energy levels: one for a one-electron system, one for a two-electron system.

2. RAMSEY SPECTROSCOPY (25 POINTS)

Now we turn to the ground state of the Cesium atom, which is the element on the sixth row of the first column in the periodic table. This state is being used as the definition of the second. The only stable isotope of Cesium has a nuclear spin of $I = 7/2$.

- A. Calculate the g_F factors for the two hyperfine components of the ground state.
- B. Make a sketch of the splitting of the ground state of Cesium in low magnetic fields, and label the lines according to their m_F . Then indicate in the same figure the splitting in large magnetic fields, and show the connection between the two regimes.
- C. Indicate in the figure that you just made which states are low-field seeking and which states are high-field seeking.
- E. Draw a schematic diagram of a Ramsey setup making use of separated oscillatory fields. Label the essential components, and indicate the relevant quantum numbers (F, m_F) for Cesium atoms in various parts of the setup. Also draw the typical spectrum that is obtained.
- F. Explain how this experiment works using pictures of a Bloch sphere with the state vector that describes the state of the Cesium atom. Use these pictures to explain the minima and maxima in the spectrum that is obtained.
- H. Explain how the apparatus described in E benefits from the use of cold atoms, and give the reasons why such a machine is better suited as a frequency standard compared to a Hydrogen maser.

3. NON-DESTRUCTIVE PHOTON DETECTION (20 POINTS)

We start with an atom that has 3 states labelled $|i\rangle$, $|g\rangle$ and $|e\rangle$. There is a cavity, with or without a photon, that is tuned to resonance with the transition $|g\rangle \leftrightarrow |e\rangle$. In this question we investigate how we can use the atoms to probe whether a photon is present in the cavity, without destroying the photon.

- A.** The atom has to be prepared in a superposition of states for this trick to work. Which states are coupled, and how is this done? Give the initial atomic wave function and the wave function after preparation.
- B.** What kind of interaction of the atom and the photon can take place in the cavity that can be detected but does not lead to destruction of the photon? Give the wave function after passage through the cavity if the photon is present, and if it is not.
- C.** How does the final detection step work? Give the wave function following the detection step if the photon is present and if it is not.

4. MOLECULAR SPECTRA (15 POINTS)

- A.** Make a schematic drawing of the first four rotational levels in a vibrational ground state $v'' = 0$ and the first excited state $v' = 1$.
- B.** Assuming these vibrational states to be in the same electronic potential, indicate the allowed transitions between these rotational levels, and give these transitions the appropriate labels.
- C.** How does the situation change if the vibrational states are in different electronic states? Indicate possible new transitions, and label them.
- D.** What is the reason for the frequency difference between the P, Q and R branch? Which approximation is therefore not valid here?

5. DIATOMIC MOLECULES IN AN EXTERNAL MAGNETIC FIELD (15 POINTS)

- A.** Give a drawing of the relevant vectors and their projections (\vec{J} , \vec{R} , Ω , Λ) for a Hund's case (a) molecule.
- B.** Give the magnetic moment μ_{Ω} of these molecular states: a) $^1\Pi$ b) $^3\Sigma$ c) $^2\Pi_{3/2}$ d) $^2\Pi_{1/2}$
- C.** Without a magnetic field, the magnetic moment μ_{Ω} precesses around \vec{J} to form a magnetic moment μ_J . In an external field, \vec{J} precesses around \vec{B} . Knowing this, derive the effective time-averaged value of the magnetic dipole moment of a rotating molecule in an external magnetic field, μ_{eff} .

